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$1/m_b$ Corrections in the ACCMM Model for Inclusive Semileptonic B Decay

Grant Baillie*

Department of Physics, University of California, Los Angeles

405 Hilgard Avenue, Los Angeles, CA, 90024-1547, USA

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Abstract

We re-examine the model of Altarelli, Cabibbo, Corbò, Maiani and Martinelli for inclusive semileptonic B decay, in the light of recent calculations in heavy quark effective theory. The model can be shown to have no $1/m_b$ corrections, with a suitable definition of the b quark mass m_b. However, we find that the structure of the $1/m_b^2$ terms is incompatible with the predictions of heavy quark effective theory. The numerical significance of this discrepancy is discussed.

*E-mail address: baillie@physics.ucla.edu

I. INTRODUCTION

Recently, there has been much progress in the application of heavy quark effective theory (HQET) to the calculation of inclusive semileptonic decays of B hadrons. It was shown by Chay, Georgi and Grinstein [1] that the problem can be attacked using methods familiar from deep inelastic scattering. In particular, one can perform an operator product expansion on the hadronic tensor contributing to $H_b \to X \ell \bar{\nu}_\ell$, where H_b represents a (heavy) bottom hadron, and X a sum over final hadron states. At leading order in this expansion, the parton model prediction results, while higher-order terms, which are suppressed by inverse powers of the b quark mass m_b, give non-perturbative corrections. In addition, it was demonstrated the expansion is free of $1/m_b$ terms, essentially because in the effective theory, the operators with the correct dimensions to contribute to such terms have matrix elements which vanish to leading order.

The $1/m_b^2$ corrections to the parton model for the inclusive differential rate $d\Gamma/dE_\ell$ (where E_ℓ is the final lepton energy in the B hadron center of mass) were calculated by Bigi *et al* [2]. The non-perturbative corrections enter the calculation in the form of matrix elements of higher dimension operators: the two that contribute in this case are

$$\frac{1}{m_b^2} \left\langle H_b(v) \left| \bar{b}_v \, (v \cdot D)^2 \, b_v \right| H_b(v) \right\rangle \qquad \text{and} \qquad \frac{g}{m_b^2} \left\langle H_b(v) \left| \bar{b}_v \, \sigma_{\alpha\beta} \, G^{\alpha\beta} \, b_v \right| H_b(v) \right\rangle$$

where $H_b(v)$ denotes a B hadron with velocity v, D is the QCD covariant derivative, $G_{\alpha\beta}$ is the QCD field strength tensor and b_v denotes the quark field in the heavy quark effective theory.

Subsequently, a very detailed exposition of the $1/m_b^2$ corrections was given by Manohar and Wise [3]. This paper presented double differential distributions (which also appeared around the same time in [4]), and also considered polarisation effects in Λ_b decays. In addition, it was also shown that some of the higher order terms could be interpreted as resulting from averaging the zeroth-order (*i.e.* parton model) decay rate over the residual 4-momentum of the bound b quark inside the B hadron.

This interpretation is reminiscent of the older calculation of Altarelli *et al* [6], which we henceforth refer to as the the ACCMM model. In this model, bound state corrections are incorporated by introducing a wavefunction for the two quarks inside the B meson, and then averaging the partonic decay rate over their relative 3-momentum. The size of the ACCMM wavefunction is determined by the Fermi momentum p_F of the quarks inside the B meson, which is of order $\Lambda_{\rm QCD}$. Consequently, one can perform a large m_b expansion of the ACCMM result, and investigate whether it can, with suitable choice of parameters, be made to agree with the QCD predictions of [1–4]. This procedure is the main focus of this letter. Throughout, we ignore the corrections of perturbative QCD to the process, discussing bound state corrections only. In Section II, we introduce our notation and briefly review the averaging of Manohar and Wise. The ideas behind the ACCMM model are introduced in Section III, where we also attempt to find a correspondence with the newer calculation to order $1/m_b^2$. We present our conclusions in Section IV.

II. AVERAGING IN HQET

From the charged-current interaction in the Standard Model

$$\begin{aligned} \mathcal{L} &= -\frac{4G_F}{\sqrt{2}}\, V_{jb}\, \bar{q}_j \gamma^\mu P_L b\;\; \bar{\ell}\gamma_\mu P_L \nu_\ell \quad + \quad \text{h.c.} \\ &\equiv -\frac{4G_F}{\sqrt{2}}\, V_{jb}\, J_j^\mu\; J_{\ell\mu} \quad + \quad \text{h.c.,} \end{aligned} \tag{1}$$

(where j is a flavour index, V is the CKM quark mixing matrix, G_F is the Fermi weak decay constant and P_L is the projection operator $(1-\gamma^5)/2$) one can derive the inclusive differential semileptonic decay rate

$$\frac{d\Gamma}{dq^2 dE_\ell dE_\nu} = \frac{G_F^2}{4\pi^3}\, |V_{jb}|^2\; L_{\alpha\beta}\, W^{\alpha\beta}. \tag{2}$$

Here one has summed over all final states X_j containing a quark of flavour $j = u$ or c. The 4-vector q is just the sum of the 4-momenta p and p' of the final state lepton and antineutrino, while E_ℓ and E_ν denote the energies of these particles. Throughout, we neglect the lepton mass m_ℓ. The leptonic tensor $L_{\alpha\beta}$ is given by

$$L_{\alpha\beta} = 2\left(p_\alpha p'_\beta + p_\beta p'_\alpha - g_{\alpha\beta}\, p\cdot p' - i\,\epsilon_{\alpha\beta\mu\nu}p^\mu p'^\nu\right), \tag{3}$$

where our convention for the totally antisymmetric tensor is $\epsilon_{0123} = +1$. $W_{\alpha\beta}$ can be expressed as

$$W_{\alpha\beta} = \sum_{X_j} \delta^4\left(P_{H_b} - P_X - q\right) \left\langle H_b \left| J^\dagger_{j\alpha}(0)\right| X_j\right\rangle \left\langle X_j \left| J_{j\beta}(0)\right| H_b\right\rangle . \tag{4}$$

In this expression, we have normalised the B hadron state $|H_b\rangle$ as in ref. [3] to remove extra factors of mass from the decay rate (2). In the parton model, $W_{\alpha\beta}$ is approximated by its value from free quark decay:

$$\begin{aligned} W_{\alpha\beta} &= W^{(0)}_{\alpha\beta}\left(m_b, v\right) \\ &= \delta\left(m_b^2 - 2m_b\left(E_\ell + E_\nu\right) + q^2 - m_j^2\right) \\ &\quad \times \left\{-\tfrac{1}{2}g_{\alpha\beta}\left(m_b - E_\ell - E_\nu\right) + m_b\, v_\alpha v_\beta - \tfrac{1}{2}i\epsilon_{\alpha\beta\mu\nu}v^\mu q^\nu\right\}. \end{aligned} \tag{5}$$

Here we have introduced the velocity v_μ of H_b; in addition we explicitly note the dependence on m_b and v_μ because these will later be averaged over. As shown in reference [3], most of the $1/m_b^2$ corrections to the parton model result to $W_{\alpha\beta}$ can be obtained by averaging the RHS of equation (5) over the residual 4-momentum k_α of the bound b quark. If one defines the mass m'_b, velocity v'_μ and momentum P_μ of the moving b quark via

$$P_\mu = m'_b v'_\mu = m_b v_\mu + k_\mu\,; \quad \left(v'^2 = v^2 = 1\right), \tag{6}$$

then one can expand its contribution to $W_{\alpha\beta}$ to second order in k:

$$\frac{1}{v'_0}W^{(0)}_{\alpha\beta}\left(m'_b, v'\right) \approx \left(1 + k^\mu\frac{\partial}{\partial k^\mu} + \frac{1}{2}k^\mu k^\nu \frac{\partial^2}{\partial k^\mu \partial k^\nu}\right)\frac{1}{v'_0}W^{(0)}_{\alpha\beta}\left(m'_b, v'\right)\bigg|_{k=0} . \tag{7}$$

Note that this expression contains an expansion of the δ function in (5), which cannot be justified everywhere in the Dalitz plot. In fact, the OPE contains an analagous expansion (of the j-quark propagator), which breaks down in some of the regions where a partonic description is not reasonable [5]. We will assume throughout this paper that we are in a region of phase space where this truncation is valid.

It turns out that almost all the terms coming from the OPE analysis up to order $1/m_b^2$ can be reproduced by replacing $W_{\alpha\beta}$ by the average (over k) of equation (7), with the average values

$$\langle k^\alpha \rangle = E_b m_b v^\alpha \tag{8a}$$

$$\left\langle k^\alpha k^\beta \right\rangle = -\frac{2}{3}\, m_b^2 \left(g^{\alpha\beta} - v^\alpha v^\beta \right). \tag{8b}$$

From this point of view, the order of magnitude of the quantities E_b and K_b might not be obvious; however, they can be expressed as matrix elements of operators between B meson states. As a result, they are expected to be of order $(\Lambda_{\rm QCD}/m_b)^2$. Incidentally, one should note that the corrections which cannot be accounted for via this averaging are proportional to the parameter G_b, which is defined in HQET by

$$G_b = \frac{Z_b}{4m_b^2} \left\langle H_b\,(v,s) \right| \bar{b}_v\, gG_{\alpha\beta}\, \sigma^{\alpha\beta}\, b_v \left| H_b\,(v,s) \right\rangle, \tag{9}$$

where s and v denote the spin and velocity of the heavy meson H_b, and Z_b is a renormalisation constant. To leading order in $1/m_b$, this operator is a spin-spin interaction between the heavy quark and its surrounding "brown muck". A simple physical interpretation of its effects has not yet been given, however.

III. COMPARISON WITH ACCMM

The ACCMM model introduces bound state corrections to the parton result for B meson decay by considering the meson to be made up of a b and spectator quarks. Furthermore, the energy of the spectator is assumed to be given in terms of its 3-momentum $\mathbf{p}_{sp}$ by

$$E_{sp} = \sqrt{\mathbf{p}_{sp}^2 + m_{sp}^2}. \tag{10}$$

where m_{sp} is a free parameter of order $\Lambda_{\rm QCD}$. The 4-momentum P^μ of the b quark is now fixed by the requirement that the sum of the b and spectator momenta be that of the B hadron. One finds that

$$
\begin{aligned}
P_0 &= M_B - E_{sp} = M_B - \sqrt{\mathbf{p}_{sp}^2 + m_{sp}^2} \\
\mathbf{P} &= -\mathbf{p}_{sp}
\end{aligned}
\tag{11}
$$

and hence that the square of the invariant mass of the moving b quark is just

$$
m_b'^2 = P^2 = M_B^2 + m_{sp}^2 - 2M_B\sqrt{\mathbf{p}_{sp}^2 + m_{sp}^2}. \tag{12}
$$

To perform the averaging over the motion of the quarks inside the hadron, one introduces a wavefunction $\psi(\mathbf{p}_{sp})$. Typically, one expects that this function will have a width of order $\Lambda_{\rm QCD}$ and that it will fall off rapidly for values of momentum $\mathbf{p}_{sp}$ larger than this value. Also, because we understand the B meson to be an L=0 state, we assume that $\psi(\mathbf{p}_{sp})$ is spherically symmetric. In practice, one takes

$$
|\psi(\mathbf{p}_{sp})|^2 = N \, \exp\left(\mathbf{p}_{sp}^2/p_F^2\right), \tag{13}
$$

where N is a normalisation factor and p_F, the Fermi momentum of the quarks inside the hadron, is taken to be an adjustable parameter of order $\Lambda_{\rm QCD}$. One can now write down the hadronic tensor in the model, by assuming it is just the average over the quark 3-momentum of the boosted free quark value:

$$
W_{\alpha\beta} = \int d^3\mathbf{p}_{sp} \, |\psi(\mathbf{p}_{sp})|^2 \, \frac{1}{v_0'} W_{\alpha\beta}^{(0)}\left(m_b', v' = P/m_b'\right). \tag{14}
$$

In this expression, we have explicitly included a Lorentz time-dilation factor in the integral, which was omitted in [6]. It should also be noted that in [6] the integral is cut off at the maximum value of $|\mathbf{p}_{sp}|$ allowed in the decay; however, the wavefunction is so small at this point (the exponential factor in (13) is tiny there) that we can ignore the effect of this upper limit in what follows.

Since the wavefunction factor in (14) effectively restricts $|\mathbf{p}_{sp}|$ to be at most of order p_F, we can envisage expanding the factor $W_{\alpha\beta}^{(0)}(m_b', v')/v_0'$ in (14) by treating $\mathbf{p}_{sp}$, p_F and m_{sp} as small. Clearly, the first term in this expansion will just be $W_{\alpha\beta}^{(0)}(M_B, v)$. The fact that this depends on M_B and not m_b is not accidental: the ACCMM model was specifically constructed to try to circumvent the dependence of the partonic decay rate on the unknown

parameter m_b. Just as was the case in HQET, our expansion will contain average values, which are now given (as averages over $\mathbf{p}_{sp}$) by

$$\langle f \rangle = \int d^3\mathbf{p}_{sp} \, |\psi(\mathbf{p}_{sp})|^2 \; f(\mathbf{p}_{sp}). \tag{15}$$

If the two types of averaging were to produce identical results to this order, we would be able to express the HQET parameters m_b, E_b and K_b in terms of M_B, p_F and m_{sp}. In fact, there would be some dependence on the ACCMM wavefunction through, for example, a quantity like $\langle \mathbf{p}_{sp} \rangle$. Because our expansion starts at M_B, and because the HQET expansion contains no $1/m_b$ terms, we will evidently have to absorb $1/M_B$ terms into the definition of the quark mass m_b. Evidently, in ACCMM we have from (11)

$$\langle P^\alpha \rangle = v^\alpha \left(M_B - \left\langle \sqrt{\mathbf{p}_{sp}^2 + m_{sp}^2} \right\rangle \right). \tag{16}$$

Consistency with (6) and (8a) can only be obtained if

$$m_b \, (1 + E_b) = M_B - \left\langle \sqrt{\mathbf{p}_{sp}^2 + m_{sp}^2} \right\rangle. \tag{17}$$

The 4-momentum k in (6) can now be expressed in the B rest frame as

$$k^\mu = \left(E_b M_B - \sqrt{\mathbf{p}_{sp}^2 + m_{sp}^2} + \left\langle \sqrt{\mathbf{p}_{sp}^2 + m_{sp}^2} \right\rangle , \, -\mathbf{p}_{sp} \right), \tag{18}$$

so that we have, to leading nonvanishing order

$$\begin{aligned} \left\langle k^\alpha k^\beta \right\rangle = & \left(\left\langle \mathbf{p}_{sp}^2 + m_{sp}^2 \right\rangle - \left\langle \sqrt{\mathbf{p}_{sp}^2 + m_{sp}^2} \right\rangle^2 \right) v^\alpha v^\beta \\ & - \tfrac{1}{3} \left\langle \mathbf{p}_{sp}^2 \right\rangle \left(g^{\alpha\beta} - v^\alpha v^\beta \right). \end{aligned} \tag{19}$$

On the RHS of this equation, the first term comes from the 00-component in the B rest frame, while the second is the ij-th, taking into account rotational symmetry. The $0i$- and $i0$-components can easily be seen to vanish.

Since the two tensors in (19) are linearly independent, the only way to reconcile this equation with (8b) is to have

$$K_b = \frac{\left\langle \mathbf{p}_{sp}^2 \right\rangle}{2 m_b^2} \tag{20}$$

and also for the coefficient of $v^\alpha v^\beta$ to be of order $\Lambda^3_{\rm QCD}/m_b$, *i.e.* to vanish to the order we are considering. However, for a general wavefunction of size p_F, this quantity will be of order p_F^2. In addition, one can compute the contribution of the extra term to the quantity $L_{\alpha\beta}W^{\alpha\beta}$ in (2), to make sure it does not vanish. We find a contribution to this quantity of

$$\begin{aligned} 2A_b m_b^2 E_\nu \ \{\delta'(X)\ (6m_b E_\ell - 4E_\ell^2 - 4E_\ell E_\nu - q^2) \\ + 2\ \delta''(X)\ (m_b - E_\ell - E_\nu)^2\,(2m_b E_\ell - q^2)\Big\} \\ \text{with } X \equiv m_b^2 - 2m_b\,(E_\ell + E_\nu) + q^2 - m_j^2, \end{aligned} \tag{21}$$

where we have defined the dimensionless parameter A_b by

$$A_b\, m_b^2 \quad = \quad \left\langle \mathbf{p}_{sp}^2 + m_{sp}^2 \right\rangle - \left\langle \sqrt{\mathbf{p}_{sp}^2 + m_{sp}^2} \right\rangle^2 . \tag{22}$$

Incidentally, one should note that this contribution cannot just cancel the Lorentz time-dilation factor mentioned after (14), as that gives an overall factor $(1 - E_b)$ to the decay. Consequently, it seems to be impossible to find an exact correspondence between the averaging of the phenomenological ACCMM model and the QCD calculations of HQET.

IV. CONCLUSIONS

We have analysed, in two different approaches, the corrections to the parton model for inclusive semileptonic B meson decay up to order $1/m_b^2$. Although we have found that the ACCMM model cannot be put into exact correspondence with the HQET calculation, we should point out that the discrepancy between the two is likely to be small numerically. This is mainly due to the fact that $1/m_b$ corrections vanish in both cases if equation (17) is satisfied. As demonstrated in [3], the $1/m_b^2$ terms adjust the parton result by around a percent in the region where the expansion makes sense. Also, the additional term appearing in $\langle k^\alpha k^\beta \rangle$ (cf. equation (19)) in the ACCMM case is likely to be suppressed compared to the other term for a monotonically decreasing wavefunction. This term would dominate the other corrections in the $m \to \infty$ limit, but for the B it is practically indistinguishable from the higher order terms we have ignored.

The ACCMM model's disagreement can be traced to its assumption that the spectator quark in the B meson can be treated as an on-shell particle of mass m_{sp}, of the order of the QCD scale. This led to one having to define m_b in such a way so as to avoid $1/m_b$ terms in the expansion. However, making this definition was not sufficient to ensure that the $1/m_b^2$ terms had the correct form. In the context of HQET, it is clear that the "spectator quark" of ACCMM corresponds to "brown muck". However, it seems unlikely that the latter can be treated simply as an object of mass m_{sp}. If one wanted to improve the model to make it compatible with HQET, one could start by relaxing the on-shell condition for the spectator quark.

Finally, it is worth noting that equations (6) and (8), together with (18) and the way we defined the quark mass in ACCMM, suggest that, as far as these inclusive processes are concerned, the parameter E_b can be absorbed into a redefinition of m_b. This can be confirmed by examining the expressions in section 6 of reference [3]: the E_b correction terms have the form

$$E_b m_b \frac{\partial}{\partial m_b} \text{(parton model quantity)},$$

where the partial derivative is taken at fixed lepton momenta. Presumably, this should still hold true in the case of τ decay modes, where the lepton masses cannot be ignored [7]. This observation might be useful if one were trying to fit the HQET parameters to an experimental spectrum, for example.

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